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ELECTRODYNAMIC PROCESSES IN A SURFACE LAYER IN MAGNETOABRASIVE POLISHING

N. N. Grinchik,[a] **O. P. Korogoda,**[b] **and N. S. Khomich**[b]

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A consistent physicomathematical model of the propagation of an electromagnetic wave in a heterogeneous medium has been constructed with the use of the generalized wave equation and Dirichlet theorem. Twelve conditions at the interfaces between adjoining media were obtained and substantiated without using, in an explicit form, the surface charge and surface current. The conditions are fulfilled automatically in each section of the heterogeneous medium and are conjugate, thus making it possible to use schemes of through counting for calculations. The effect of the concentration of "medium-frequency" waves with length of the order of hundreds of meters at the fractures and wedges of domains of size 1–3 μm has been established for the first time.

Keywords: *polishing, electromagnetic field, electric double layer, induced surface charge, domains, vacancies, heterogeneous medium.*

Introduction. During the interaction of an external magnetic field and a magnetorheological suspension, the particles are magnetized, and magnetic dipoles with the moment oriented predominantly along the field are formed. "Chains" along the force lines of the field [1–7] appear that periodically act on the processable surface with a frequency $\omega = l/v$. A fixed elemental area of the material periodically experiences the effect of the magnetic field of one direction. Actually, the frequency and duration of the pulse will be still higher because of the rotation of the magnetoabrasive particle due to the presence of the moment of forces on contact and of the friction of the particle against the processable part. In what follows, we will not take into account the effect of rotation.

We assume that the particle velocity on the polisher is v. If the particle radius is r, then the angular frequency is $\omega = 2\pi v/r$, and precisely this frequency determines the frequency of the effect of the variable magnetic field component due to the fact that for a ferromagnetic $\mu > 1$. The magnetic permeability μ of ferromagnetics, which are usually used in magnetoabrasive polishing, is measured by thousands of units in weak fields. However, in polishing, the constant external magnetic field is strong and amounts to 10^5–10^6 A/m, and in this case the value of μ for compounds of iron and nickel and for Heusler alloy decreases substantially.

Because of the presence of a strong external magnetic field H_0 the "small" absolute value of μ of an abrasive particle leads to a periodic "increase" and "decrease" in the normal component of the magnetic induction near the processable surface. In the present work we used neodymium magnets (neodymium–iron–boron) with H_0 = 485,000 A/m. The magnetic permeability of a magnetoabrasive particle based on carbonyl iron was assumed in this case to be equal to μ_1 = 4.2.

Due to the continuity of the normal magnetic induction component $B_{n1} = B_{n2}$, where $B_{n1} = \mu_1\mu_0 H_1$; $B_{n2} = \mu_2\mu_0 H_2$. For example, in glasses $\mu_2 = 1$; therefore at the boundary of contact of the glass with the magnetoabrasive particle an additional variable magnetic field of strength $H_1 > H_0$ appears.

In [8–10], magnetic field-induced effects in silicon are considered: a nonmonotonic change in the crystal lattice parameters in the surface layer of silicon, the gettering of defects on the surface, the change in the sorption properties of the silicon surface, and the change in the mobility of the edge dislocations and in the microhardness of silicon.

In [11–17], the influence of an electromagnetic field on the domain boundaries, plasticity, strengthening, and on the reduction of metals and alloys was established.

[a]A. V. Luikov Heat and Mass Transfer Institute, National Academy of Sciences of Belarus, 15 P. Brovka Str., Minsk, 220072, Belarus; email: nngrin@yandex.ru; [b]Polimag Unitary Enterprise, 77 Partizanskii Ave., Minsk, 220107, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 83, No. 3, pp. 598–607, May–June, 2010. Original article submitted April 2, 2009; revision submitted June 10, 2009.

In view of the foregoing, it is of interest to find the relationship between the discrete-impulse action of a magnetic field of one direction on the surface layer of the processable material that contains domains. According to [1, p. 9], the size of domains is as follows: 0.05 μm in iron, 1.5 μm in barium ferrite; 8 μm in the MnBi compound, and 0.5–1 μm in the acicular gamma ferric oxide. According to [5, p. 7], the size of a domain may reach 10^6 cm^3 (obtained by the method of magnetic metallography).

As a rule, an abrasive exhibits a distinct shape anisotropy, whereas the frequency of the effect is determined by the concentration of abrasive particles in a hydrophobic solution and by the velocity of its motion. We assume that on the surface of a processable crystal the magnetic field strength $H(t) = H_1 \sin^4 (\omega t) + H_0$.

It is required to find the value of the magnetic field strength in the surface layer that has the characteristics λ_1, ε_1, and μ_1 and contains domains with the electrophysical properties λ_2, ε_2, and μ_2. The domains may have the form of a triangular prism, a bar, a cylinder, etc.

Mathematical Model. Two approaches are applicable for the solution of the problem posed. One can study in detail the effect of an electromagnetic field on electric charges that exist on their own or enter into the composition of the molecules or atoms of the medium. In this case, the needed computations are cumbersome because of the necessity of taking into account the effect exerted on each charge not only by an incident wave, but also by secondary waves from all of the remaining charges [18].

The other approach to solving the problem rests on the phenomenological electrodynamics, the premises of which serve as a basis of the investigations carried out in the present work. We will consider the interface S between two media with different electrophysical properties. Under the action of an external electromagnetic field, induced surface charges σ and surface currents $\mathbf{i}_\tau$ (the vectors lying in the plane tangential to the surface S) appear on the contact. On both sides from the interface the vectors of the magnetic field strength **H** and of magnetic induction **B**, as well as the vectors of the electric field **E** and of electric displacement **D**, are finite and continuous, but at the interface they can have a discontinuity of the first kind (discontinuity of functions).

In the investigation of an electric field that interacts with a material medium, we will use the Maxwell equations [3, 18]:

$$\mathbf{j}_{\text{tot}} = \nabla \times \mathbf{H} , \quad \nabla \cdot \mathbf{D} = \rho , \tag{1}$$

$$-\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{E} , \quad \nabla \cdot \mathbf{B} = 0 , \tag{2}$$

where $\mathbf{j}_{\text{tot}}$ and **D** are defined by the equations

$$\mathbf{j}_{\text{tot}} = \lambda \mathbf{E} + \frac{\partial \mathbf{D}}{\partial t} ; \quad \mathbf{D} = \varepsilon \varepsilon_0 \mathbf{E} ; \quad B = \mu \mu_0 H .$$

On the surface S the system of equations is augmented by the well-known conditions [3, 18]:

$$D_{n1} - D_{n2} = \sigma , \tag{3}$$

$$E_{\tau 1} - E_{\tau 2} = 0 , \tag{4}$$

$$B_{n1} - B_{n2} = 0 , \tag{5}$$

$$\mathbf{H}_{\tau 1} - \mathbf{H}_{\tau 2} = \mathbf{j}_\tau \times \mathbf{n} . \tag{6}$$

The symbols n and τ designate the normal and tangential (to the surface S) vector components, whereas the subscripts 1 and 2 designate adjoining media with different electrophysical properties. It should be noted that τ can denote any direction tangential to the discontinuity surface.

The value of the surface charge and the structure of the electric double layer can be attributed to different factors: in the case of the electrolyte–metal contact — to the transition of ions from an electrode into a solution, as well as to the specific adsorption of ions of one sign on the electrode surface and to the orientation of polar molecules near the electrode surface [2]; the structure of the double electric double layer when two solid conductors or a dielectric and conductor come into contact is caused by other reasons and has its specific features [7, 19].

The structure of the electric double layer exerts a substantial influence on the electrokinetic phenomena, the rate of electrochemical processes, and on the stability of colloidal systems. In view of the indicated reasons, the electric double layer causes great difficulties in modeling electromagnetic fields in a laminated medium. The construction of equivalent substitution schemes for taking into account the electric double layer by means of introducing the surface capacity [20] found experimentally makes sense only for the range of experimental conditions.

In optics and radiophysics [6, 21–43], to take into account the characteristic features of the electric double layer, a matrix of impedances is assigned at the interfaces; the matrix is determined experimentally or, in some cases, theoretically on the basis of quantum notions [26, 30, 34–46]. According to [2], the surface charge not only characterizes the properties of the surface, but is also a function of the process, i.e., $\sigma(\mathbf{E}(\partial \mathbf{E}/\partial t), \mathbf{H}(\partial \mathbf{H}/\partial t))$, therefore the surface impedances [6, 21–43] are valid for the conditions under which they were determined and are not used under other experimental conditions.

We will show that σ can be calculated from the Maxwell phenomenological macroscopic equations of an electromagnetic field and from the electric charge conservation law that takes into account the specifics of the interface between adjoining media. We note that in Eq. (6) the values for the surface currents $\mathbf{i}_\tau$ were not determined, and there are no closing relations for them.

We will formulate a physicomathematical model of the propagation of electromagnetic waves in a laminated medium. The media in contact are considered homogeneous. We operate with rot on the left- and right-hand sides of the first equation for the total current (1) and multiply by $\mu_0\mu$; then we differentiate the second equation in (2) with respect to time. Taking into consideration the solenoidality of the magnetic field (2) and the rule of the repeated application of the operator ∇ to the vector $\mathbf{H}$, we obtain

$$\varepsilon\varepsilon_0 \frac{\partial^2 \mathbf{H}}{\partial t^2} + \lambda\mu_0 \frac{\partial \mathbf{H}}{\partial t} = \frac{1}{\mu} \nabla^2 \mathbf{H} . \tag{7}$$

In the Cartesian coordinates Eq. (7) will have the form

$$\begin{aligned}
\varepsilon\varepsilon_0 \frac{\partial^2 H_x}{\partial t^2} + \lambda\mu_0 \frac{\partial H_x}{\partial t} &= \frac{1}{\mu}\left(\frac{\partial^2 H_x}{\partial x^2} + \frac{\partial^2 H_x}{\partial y^2} + \frac{\partial^2 H_x}{\partial z^2}\right), \\
\varepsilon\varepsilon_0 \frac{\partial^2 H_y}{\partial t^2} + \lambda\mu_0 \frac{\partial H_y}{\partial t} &= \frac{1}{\mu}\left(\frac{\partial^2 H_y}{\partial x^2} + \frac{\partial^2 H_y}{\partial y^2} + \frac{\partial^2 H_y}{\partial z^2}\right), \\
\varepsilon\varepsilon_0 \frac{\partial^2 H_z}{\partial t^2} + \lambda\mu_0 \frac{\partial H_z}{\partial t} &= \frac{1}{\mu}\left(\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2}\right).
\end{aligned} \tag{8}$$

On the interface the following relation is also valid [3]:

$$\operatorname{div} \mathbf{i} + I_{qx_1} - I_{qx_2} = -\frac{\partial \sigma}{\partial t} . \tag{9}$$

Conditions (3)–(6) will be written in the Cartesian coordinates system:

$$D_{x_1} - D_{x_2} = \sigma , \tag{10}$$

$$E_{y_1} - E_{y_2} = 0\,, \tag{11}$$

$$E_{z_1} - E_{z_2} = 0\,, \tag{12}$$

$$B_{x_1} - B_{x_2} = 0\,, \tag{13}$$

$$H_{y_1} - H_{y_2} = i_z\,, \tag{14}$$

$$H_{z_1} - H_{z_2} = i_y\,, \tag{15}$$

$\mathbf{i}_\tau = i_y\mathbf{j} + i\mathbf{k}$ is the density of the surface current; in this case the x coordinate is directed along the normal to the interface. The density i_y, i_z of the surface currents is understood to be the quantity of electricity flowing per unit time through a unit length of the segment located on the surface through which the current flows and perpendicular to the current direction.

The order of the system of differential equations (8) is equal to 18. Therefore at the interface S one should, generally speaking, assign nine boundary conditions. Moreover, ay this interface conditions (13)–(15) containing unknown (prior to solution) quantities should also be satisfied. Consequently, the general number of conjugation conditions at the interface S must be equal to 12 for correct solution of the problem.

Differentiating expression (10) with respect to time and taking into account relation (9), at the interface between the media we obtain an equation for the normal components of the total current:

$$\operatorname{div}\mathbf{i}_\tau + \mathbf{j}_{\mathrm{tot}\,x_1} = \mathbf{j}_{\mathrm{tot}\,x_2}\,, \tag{16}$$

which makes it possible to exclude from consideration the surface density of the charge σ. For an arbitrary function f we introduce the notation $[f]\,|_{x=\xi} = f_1\,|_{x=\xi+0} - f_2\,|_{x=\xi-0}$. Then expression (16) takes the form

$$[\operatorname{div}\mathbf{i}_\tau + \mathbf{j}_{\mathrm{tot}\,x}]\,|_{x=\xi} = 0\,. \tag{17}$$

The system of equations (8) has been formulated by us only for the magnetic field strength vector; therefore from the conditions at the interfaces between adjoining media (10)–(15) we must exclude the electric field strength. From Eq. (1) and condition (17) it is evident that the following relation is satisfied:

$$[\operatorname{div}\mathbf{i}_\tau + (\operatorname{rot}\mathbf{H})_x]\,|_{x=\xi} = 0\,. \tag{18}$$

We will differentiate conditions (13)–(15) for the magnetic induction and magnetic field strength with respect to time. Assuming that $\mathbf{B} = \mu\mu_0\mathbf{H}$, we obtain

$$\left[\frac{\partial B_x}{\partial t}\right]\Bigg|_{x=\xi} = 0\,,\quad \left[\frac{1}{\mu\mu_0}\frac{\partial B_y}{\partial t}\right]\Bigg|_{x=\xi} = \frac{\partial i_z}{\partial t}\,,\quad \left[\frac{1}{\mu\mu_0}\frac{\partial B_z}{\partial t}\right]\Bigg|_{x=\xi} = \frac{\partial i_y}{\partial t}\,. \tag{19}$$

Due to the equality of the tangential projections of the electric field along z and y, according to conditions (11) and (12), the expressions for the densities of the surface current i_z and i_y have the form

$$i_z = \bar{\lambda}E_z\,|_{x=\xi}\,,\quad i_y = \bar{\lambda}E_y\,|_{x=\xi}\,, \tag{20}$$

where

$$\bar{\lambda} = \frac{1}{2}(\lambda_1 + \lambda_2)\big|_{x=\xi} \tag{21}$$

is the average value of the electrical conductivity at the interfaces between adjoining media in accordance with the Dirichlet theorem for the piecewise-smooth piecewise-differentiable function [47].

Having multiplied the left- and right-hand sides of conditions (11) and (12) by λ, we obtain

$$[i_y]\big|_{x=\xi} = 0\,, \quad [i_z]\big|_{x=\xi} = 0\,, \tag{22}$$

i.e., at the interfaces the equality of surface currents i_y, i_z, as well as of their derivatives with respect to y and z, is satisfied.

Consequently, after the summation of conditions (22) we have

$$\left[\frac{\partial i_y}{\partial y} + \frac{\partial i_z}{\partial z}\right]\Bigg|_{x=\xi} = 0 \tag{23}$$

or

$$[\mathrm{div}\,\mathbf{i}_\tau]\big|_{x=\xi} = 0\,, \tag{24}$$

i.e., the surface divergence is equal to zero; therefore Eq. (18) will have a simpler form:

$$[(\mathrm{rot}\,\mathbf{H})_x]\big|_{x=\xi} = 0\,. \tag{25}$$

The surface current is also continuous along the coordinate x (due to the continuity of the volumetric electric charge):

$$\left[\frac{\partial i_y}{\partial x}\right]\Bigg|_{x=\xi} = 0\,, \quad \left[\frac{\partial i_z}{\partial x}\right]\Bigg|_{x=\xi} = 0\,. \tag{26}$$

With allowance for the foregoing, we have 12 relations at the interface between adjoining media; they are needed for the solution of the complete system of equations (8): classical conditions (13)–(15) for B_x, H_y, and H_z (3 conditions), nonstationary conditions for the magnetic induction vector (19) which follow from the Maxwell equations, for the surface current i_y, i_z (22), as well as for the surface divergence of the surface charge (24), for the normal component of the vortex projection of the magnetic field strength (25), as well as the condition of the continuity of derivative surface currents along the normal (26). As a whole, we have 12 conditions at the interfaces between adjoining media that are satisfied automatically for each interface, including those in the presence of strong discontinuities of the function — the magnetic field strength.

The electromagnetic phenomena appearing during incidence of plane electromagnetic waves on the interface between different media play an important role in engineering, since all the real facilities are bounded by surfaces and are inhomogeneous in space. At the same time, according to [23, pp. 687–689], investigations of the propagation of waves in a laminated conducting medium and in thin films are restricted to the computation of the coefficients of reflection and transmission, and the function $E(x)$ over the film thickness is neglected, i.e., the geometric optics approximation is employed.

With the aid of the proposed physicomathematical model it is possible to investigate the passage of an electromagnetic wave through a laminated medium not resorting to the assumptions made in [23–46].

In view of the fact that in each section of a laminated medium, conditions (16)–(26) are automatically valid and satisfied, we will use the schemes of through counting without paying attention to the interface between adjoining media. Here, it is suggested to calculate H_x at the interface as follows.

According to (13), $H_{x_1} \neq H_{x_2}$, i.e., $H_x(x)$ undergoes discontinuity of the function of the first kind, if $\mu_1 \neq \mu_2$. We will determine the value of the magnetic field strength at the discontinuity point $x = \xi$ provided that

$H_x(x)$ is a piecewise-smooth piecewise-differentiable function, i.e., has finite one-sided derivatives $H'_{x^+}(x)$ and $H'_{x^-}(x)$. In this case, at the discontinuity points x_i

$$H'_+(x_i) = \lim_{\Delta x_i \to +0} \frac{H(x_i + \Delta x_i) - H(x_i + 0)}{\Delta x_i}, \quad H'_-(x_i) = \lim_{\Delta x_i \to -0} \frac{H(x_i + \Delta x_i) - H(x_i - 0)}{\Delta x_i}. \tag{27}$$

Then, according to the Dirichlet theorem [47, pp. 255–256], the Fourier series of the function $H(x)$ at each point x, including the discontinuity point ξ, converges and its sum is equal to

$$H_{x=\xi} = \frac{1}{2}[H(\xi - 0) + H(\xi + 0)]. \tag{28}$$

The Dirichlet condition (28) has also a physical meaning. After the contact of two solid conductors, dielectrics, or electrolytes in various combinations (metal–electrolyte, dielectric–electrolyte, metal–vacuum, etc.) at the interface between adjoining media, an electric double layer is always formed the structure of which is usually unknown, but it substantially influences the electrokinetic phenomena, the rate of electrochemical processes, etc. It is important to note that in reality the electrophysical characteristics λ, ε, and $H(x)$ in the electric double layer change continuously; therefore Eq. (28) is valid for the case where the electric double layer thickness, i.e., the interface thickness, is much smaller than the characteristic dimension of a homogeneous medium. In the case of a composite, for example, a metal with inclusion of dielectric small spheres, at a large enough concentration of both components and smallness of their characteristic dimensions, overlapping of interfaces will occur, and condition (28) can be disturbed.

But if the electric double layer thickness is much smaller than the characteristic dimensions L of the objects investigated, then (28) follows also from the condition of the linear change of $H(x)$ in the region of the electric double layer. In reality the electric double layer thickness depends on the kind of containing substances and may be equal to tens of Angström units [48, p. 239]. According to the current notions, the outer lining of the electric double layer consists of two parts: the first is that formed by ions closely drawn to the metal surface (the "dense" or "Helmholz" layer of thickness h) and the second — by ions located at distances from the surface that exceed the ion radius, with the number of these ions decreasing with increase in the distance from the interface ("diffuse layer"). The potential distribution in the dense and diffuse parts of the electric double layer is in fact exponential [48], i.e., the condition of the linear character of the change in $H(x)$ is disturbed, with the sum of the charges of the dense and diffuse parts of the outer lining of the electric double layer being equal to the charge of the inner lining of the electric double layer of the metal surface. However, if the thickness h of the electric double layer is much smaller than the characteristic dimension of the object, then the expansion of $H(x)$ into a power series is valid, and we may restrict ourselves to a linear approximation. According to the more general Dirichlet theorem (1829), the physical interpretation, as well as knowledge of the functions $H(x)$ in the region of the electric double layer, is not required for the justification of (28). Nevertheless the above-indicated well-known physical characteristic features of the electric double layer confirm the validity of the fulfillment of condition (28).

The condition at the interfaces analogous to (28) for a potential field (when rot $\mathbf{H} = 0$) was also obtained in [49, p. 353] on the basis of the introduction of the surface potential, use of the Green's formula, and consideration of the double layer potential discontinuity. In [49, p. 356] it is emphasized that account for the double layer thickness and for the change in the potential within the double layer at $h/L \ll 1$ generally makes no sense; therefore it is worthwhile instead of the volumetric potential to consider the surface potential with a certain surface density. Condition (28) can be obtained from the more general Dirichlet theorem also for an eddy nonpotential field.

Thus, with allowance for the foregoing and for the validity of conditions (10)–(12) and (18)–(24) in each section of a laminated medium it is worthwhile to use the schemes of through counting for numerical solution and in this case to carry out discretization of the medium in such a way that the boundaries of the layers could have common nodes.

For numerical simulation of the propagation of electromagnetic waves in a laminated medium we used the finite-elements method for the system of equations (6)–(8) [50]. In this case the division of the medium into finite elements was made in such a way that the nodes of the finite-element grid which lie on the interface could simultaneously belong to the media with different electrophysical properties. In this case, on the interface the condition

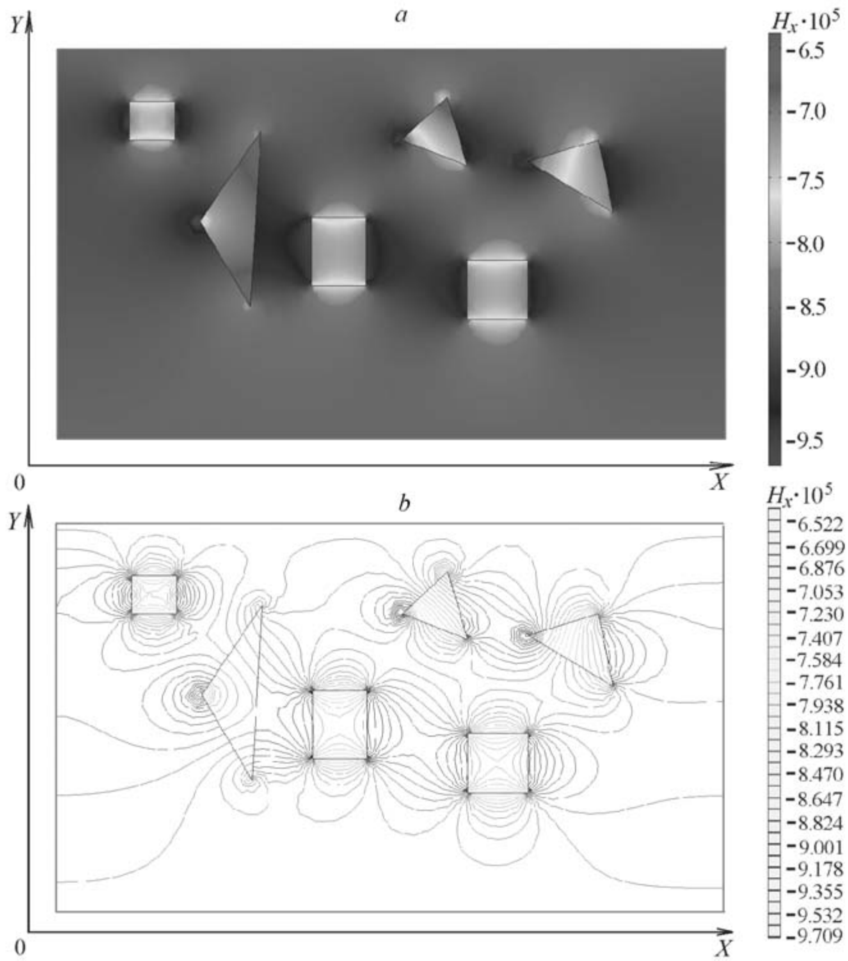

Fig. 1. Amplitude along H_x (a) and isolines (b) of the magnetic field strength.

of the equality of total currents or the equality of the charge streams should hold, if the Dirichlet condition (28) is used.

The source of an electromagnetic wave is the cyclic, parametric generation of microwaves by a moving magnetoorientated suspension of abrasive particles.

Results of Numerical Simulation. The physicomathematical model developed can be effectively used also in modeling the propagation of electromagnetic waves in media with complex geometries and strong electromagnetic field discontinuities.

The transverse cut of a cellular structure represents a set of parallelepipeds, triangular prisms of various cross sections, as depicted in Fig. 1a. An electromagnetic wave propagates across the direction of parallelepipeds and triangular prisms (channels) along the coordinate x.

The size of the investigated two-dimensional object is $14 \times 20 \cdot 10^{-6}$ m, and the sizes of the domains are 2–4 μm. The frequency of the influence of the magnetic field is $\omega = 2\pi \cdot 10^6$, and the strength of the field is

$$H_x = 21 \cdot 10^5 \sin^4 (2\pi \cdot 10^6 \, t) \ \text{A/m} . \tag{29}$$

The electrophysical properties are: of the large parallelepiped, $\mu = 1$, $\varepsilon = 8$, $\sigma = 10^{-9}$ Ω·m; of domains, $\mu = 1$, $\varepsilon = 6$, $\sigma = 10^{-8}$ Ω·m. They correspond to the electrophysical properties of glasses.

It was assumed that in a layer of thickness 15–20 μm an electromagnetic wave propagates without attenuation; therefore, on all the faces of the large parallelepiped the fulfillment of condition (29) was considered valid. On the

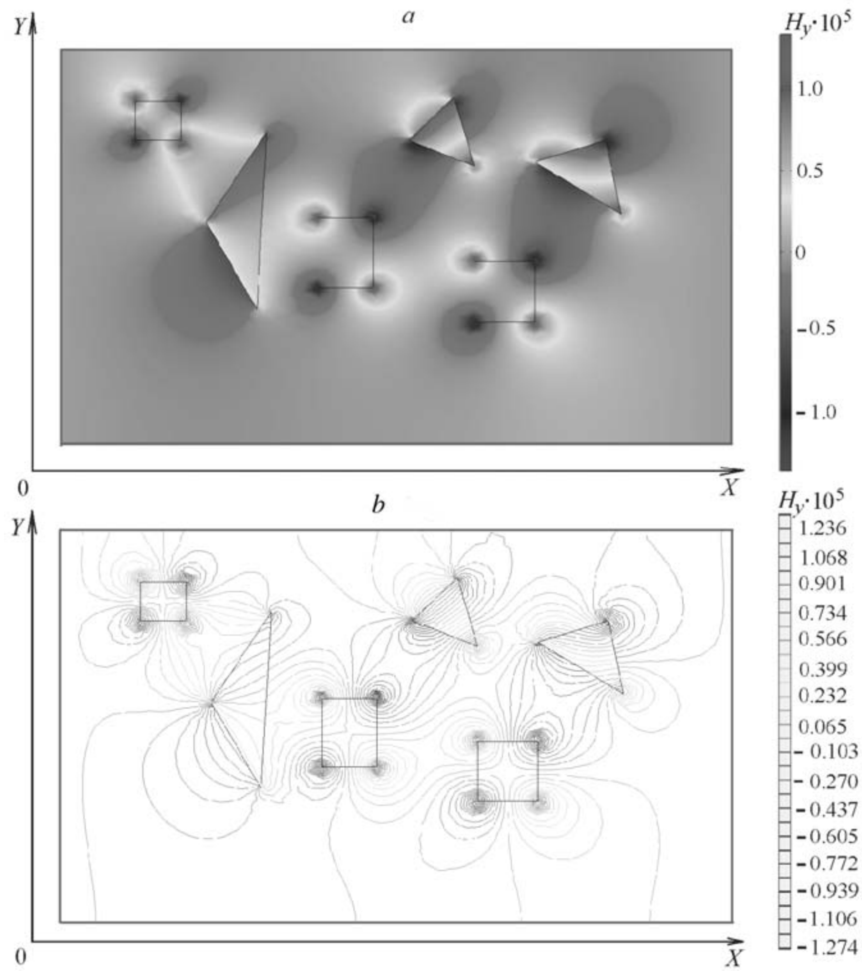

Fig. 2. Amplitude along H_y (a) and isolines (b) of the magnetic field strength.

faces of the parallelepiped that are parallel to the OX axis condition (29) corresponded to the "transverse" tangential component of the wave; on the faces parallel to OY condition (29) corresponded to the normal component of the field.

The calculations were carried out with a time step of 10^{-3} sec up to the time instant 10^{-10} sec.

Figures 1a and 2a present the amplitude values of the magnetic field strength along H_x and H_y with a comparison scale, whereas Figs. 1b and 2b present the corresponding isolines. An analysis of these figures shows that at the places of discontinuity, on the wedges, force lines of the electromagnetic field concentrate. According to [4], precisely wedges are often the sources and sinks of the vacancies that determine, for example, the hardness and plasticity of a solid body.

As is known [51], in thermodynamically equilibrium systems the temperature T and the electrical φ and chemical μ_c potentials are constant along the entire system:

$$\mathrm{grad}\, T = 0\,, \quad \mathrm{grad}\, \varphi = 0\,, \quad \mathrm{grad}\, \mu_c = 0\,.$$

If these conditions are not fulfilled (grad $T \neq 0$, grad $\varphi \neq 0$, grad $\mu_c \neq 0$), irreversible processes of the transfer of mass, energy, electrical charge, etc. appear in the system.

The chemical potential of the jth component is determined, for example, as a change of the free energy with a change in the number of moles:

$$\mu_{cj} = (\partial F / \partial n_j)_{T,V}\,, \tag{30}$$

where

$$dF = -\,SdT - PdV + HdB\,. \tag{31}$$

The last term in (31) takes into account the change in the free energy of a dielectric due to the change in the magnetic induction. The free energy of a unit volume of the dielectric in the magnetic field in this case has the form

$$F\,(T, D) = F_0 + \mu\mu_0 \frac{H^2}{2}\,. \tag{32}$$

We assume that changes in the temperature and volume of the dielectric are small. Then the mass flux is determined by a quantity proportional to the gradient of the chemical potential or, according to (31), we obtain

$$q_i = -\,D_{\mu_c}\,\mathrm{grad}\,(HdB) = -\,D_{\mu_c}\,\mathrm{grad}\,W\,,$$

where $W = \mu\mu_0 \frac{H^2}{2}$ is the density of the magnetic field in the unit volume of the dielectric.

In magnetic-abrasive polishing on the sharp protrusions of domains the gradients of magnetic energy are great, which can lead to the origination of vacancy flows.

An analysis of the results shows that the nonstationary component of the full electromagnetic energy is also concentrated in the region of fractures and wedges, i.e., at the sharp angles of domains, which may lead to the improvement of the structure of the sublayer of the treated surface due to the "micromagnetoplastic" effect. Maximum values of the nonstationary part of the total electromagnetic energy $W_{\max}$ in the sublayer correspond to a maximum value of the function $\sin^4\,(2\pi \cdot 10^6 t)$ and occur for the time instants $t = (n/4)\cdot 10^{-6}$ sec, where n is the integer, with the value of $W_{\max}$ for a neodymium magnet and a magnetoabrasive particle on the basis of carboxyl iron amounting to a value of the order of $(5–6)\cdot 10^6$ J/m^3. Having multiplied $W_{\max}$ by the volume of a domain, vacancy, or atom, we may approximately obtain the corresponding energy. The density of the electromagnetic energy in all of the cases is much smaller than the bonding energy of atoms, 10^{-18}–10^{-19} J. However, a periodic change in the magnetic field in one direction leads to a ponderomotive force that may influence the motion of various defects and dislocations to create a stable and equilibrium structure of atoms and molecules in magnetoabrasive polishing and, in the long run, in obtaining a surface with improved characteristics due to the "micromagnetoplastic" effect.

Conclusions. A coordinated physicomathematical model of the propagation of an electromagnetic wave in a heterogeneous medium has been constructed using the generalized wave equation and the Dirichlet theorem. Twelve conditions at the interfaces of adjoining media were obtained and justified without using a surface charge and surface current in an explicit form. The conditions are fulfilled automatically in each section of the heterogeneous medium and are conjugate, which made it possible to use throughput counting schemes for calculations. For the first time the effect of concentration of "medium-frequency" waves with a length of the order of hundreds of meters at the fractures and wedges of domains of size 1–3 μm has been established. Numerical calculations of the total electromagnetic energy on the wedges of domains were obtained. It is shown that the energy density in the region of wedges is maximum and in some cases may exert an influence on the motion, sinks, and the source of dislocations and vacancies and, in the final run, improve the near-surface layer of glass due to the "micromagnetoplastic" effect.

NOTATION

B, magnetic induction, Wb/m^2; **D**, electric displacement, C/m^2; **E**, electric field strength, V/m; F, free energy of the unit volume of a dielectric, J/m^3; F_0, free energy of a dielectric in the absence of a field, J/m^3; **H**, magnetic field strength, A/m; h, thickness of the electric double layer, m; I_{qx_1}, I_{qx_2}, normal components of the conduction current in media 1 and 2, C/(m^2·sec); $\mathbf{i}_\tau$, surface current, A/m; **i**, **j**, **k**, unit vectors of the orthonormalized basis; $\mathbf{j}_{\mathrm{tot}}$, total current, A/m^2; L, size of a specimen, m; l, average distance between ferroparticles, m; **n**, unit vector normal to the interface; P, pressure, Pa; q, mass flux, kg/(m^2·sec); r, radius of a particle, μm; S, interface between adjoining media; T, temperature, ^{o}C; t, time, sec; V, volume, m^3; v, velocity of the "slip" of a ferroparticle over the treated surface, m/sec; W, magnetic field density, J/m^3; x, y, z, Cartesian coordinates; ε, relative permeability; ε_0, electric constant equal to $8.58 \cdot 10^{-12}$ F/m; φ, electric potential, V; λ, electric conductivity, $\Omega \cdot$m; $\bar{\lambda}$, average value of electric conductivity, $\Omega \cdot$m; μ, relative permeability; μ_0, magnetic constant equal to $4\pi \cdot 10^{-7}$ gf/m; μ_c, chemical potential; ρ, specific electric charge, C/m^3; σ, surface density of a charge, C/m^2; ξ, point of discontinuity; ω, angular frequency, 1/sec; $\nabla \equiv \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$, symbolic operator; $[f]\,|_{x=\xi} = f_1\,|_{x=\xi+0} - f_2\,|_{x=\xi-0}$. Subscripts: 0, constant component of a magnetic field H; 1, first medium; 2, second medium; c, concentration; i, number of the grid node; j, component; max, maximum; n, τ, directions normal and tangential to the interface; x, normal component of a vector; y, z, tangential components of a vector at the interface between adjoining media; tot, total.

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